Notes on Multilevel Diffractive Optics: Part-II

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Abstract
This is the second or final part of the collection of personal notes on multilevel diffractive optics which lists a lot of unpublished MDL designs. This article provides an account of the work done during the 2016-2020 period. The structure of the manuscript is random. Some of the rudimentary diffractive optics codes used in the project is available for use at https://github.com/Sourangsu/Computational-Diffractive-Optics-Code. Furthermore, the unpublished results discussed in this manuscript along with some of the codes and designs are available for non-commercial use: https://github.com/Sourangsu/Multilevel-Diffractive-Optics-based-Computational-Photography-Code.

1. Introduction
Diffractive optics, an important part of modern optics, involves the control of optical fields by thin microstructure elements via diffraction and interference. Although the basic theoretical understanding of diffractive optics has been known for a long time, many of its applications have not yet been explored. As a result, the field of diffractive optics is old and young at the same time [1-5]. The interest in diffractive optics originates from the fact that diffractive optical elements (DOEs) are flat and lightweight. This makes their applications into compact optical systems more feasible compared to bulky refractive optics. Although these elements demonstrate excellent diffraction efficiency for monochromatic light, they fail to generate complex intensity profiles under broadband illumination. This is due to the fact that the degrees-of-freedom in these elements are insufficient to overcome their strong chromatic aberration. As a result, despite their so many advantages over refractive optics, their applications are somewhat limited in broadband systems. Recent work has shown that by harnessing structural parametric optimization of DOEs, one can design Multilevel Diffractive Lenses (MDLs) to enable multiple functionalities like achromaticity, depth of focus, wide-angle imaging, [6-46] etc. The current manuscript contains the mathematical foundations that are needed to get started. A lot of information provided herein were used by the authors themselves as a field guide/tool when doing MDL designs during the 2016-2020 period. However, besides these notes, a lot of physics still needs to be figured out to understand the topic, which is beyond the
capabilities of a single person or a small research group working within a limited time range. A few research groups have also made contributions in this area [47-50] but we will keep the discussion limited to work performed by us.

2. Results

2.1. Non-Symmetric THz MDLs

It is not necessary for MDLs to be rotationally symmetric at all. One can even design non-rotationally symmetric MDLs with almost a similar aberration corrected focusing effect as seen from the 1D and 2D THz MDL designs shown in Fig. 1 and Fig. 2.

![Fig. 1 Simulated and target 1D point-spread functions for non-symmetric 1D THz MDLs operating at (a) 0.3 THz, (b) 0.4 THz, (c) 0.5 THz and (d) 0.6 THz. The insets show the corresponding 2D line focusing at 50 mm and the pixel height distribution. Here, semi-analytic and simulated denotes the same thing and will be used interchangeably throughout this text.](image)

As it can be seen from the insets of Fig. 1 and Fig. 2, none of these MDL designs are symmetric yet the PSFs are symmetric. In terms of design parameters, the 1D MDL designs shown in Fig. 1 have total number of pixels (P) = 260, pixel width (W) = 0.1 mm, maximum pixel height (Hmax) = 1.4 mm, minimum pixel height (Hmin) = 0.1 mm, total side length (L) = 26 mm, number of levels (N) = 14, focal length (f) = 50 mm and the numerical aperture (N.A.) = 0.2516. The material used is HDPE. More discussion on the choice of the material used is discussed later.
Fig. 2 Simulated 3D point-spread functions for non-symmetric 2D THz MDLs operating at (a) 0.3 THz, (b) 0.4 THz, (c) 0.5 THz and (d) 0.6 THz. The insets show the corresponding 2D point-spread functions focusing at 50 mm, the pixel height distribution and the 1D point-spread function (PSF) plot across both the horizontal and vertical direction.

Similarly, in terms of design parameters, the 2D MDL designs shown in Fig. 2 have total number of pixels (P) = 1600, pixel width (W) = 0.65 mm, maximum pixel height (H$_{\text{max}}$) = 1.4 mm, minimum pixel height (H$_{\text{min}}$) = 0.1 mm, total side length (L) = 26 mm, number of levels (N) = 14, focal length (f) = 50 mm and the numerical aperture (N.A.) = 0.2516. The material used is HDPE. More discussion on the choice of the material used is discussed later. Due to the limitation with the lateral resolution of the 3D printer, the pixel width for the 2D MDLs was restricted to 0.65 mm instead of 0.1 mm as in the case for the 1D MDLs.

Even though the initial designs were all made in HDPE, later on the MDLs were redesigned in PLA as shown in Fig. 3. This was again due to a problem with the available 3D printers which were unable to fabricate the designs at the desired resolution. The 3D printers used were Cura Ultimaker 2+ series of printers which have reasonably good resolution amongst all commercial off-the-shelf printers available in the market. Yet still, operating these printers close to the resolution limit did not yield good results and fabricated samples were “extremely” bad. Moreover, the heat...
generated during the printing process messed with the fabricated structures. Later on, after some trial and error, PLA was chosen.

![Fig. 3 3D stl model of a (a) 1D and (b) 2D THz MDL depicting the discretization of the MDL ring/pixel height distribution. (c) Scalar diffraction propagation from the lens plane to the focal (observation) plane. (d-g) The right panels show the vertical and horizontal cross-sections of the 3D printed 1D and 2D THz MDLs.](image)

The 1D and 2D MDL shown in Fig. 3 has the same design parameters as the MDLs shown in Fig. 1 and Fig. 2 with the only difference being in the material. The optimization procedure was re-run to generate the new MDL design. Here too, the total number of pixels (P) = 260 (for 1D) and 1600 (for 2D), pixel width (W) = 0.1 mm (for 1D) 0.65 mm (for 2D), maximum pixel height (H\text{max}) = 1.4 mm, minimum pixel height (H\text{min}) = 0.1 mm, total side length (L) = 26 mm, number of levels (N) = 14, focal length (f) = 50 mm and the numerical aperture (N.A.) = 0.2516. One thing to note here is that an MDL design is too an extent sensitive to the values of the optical constants taken during the optimization. A study was carried out in [15] to specifically validate this. The MDL designs shown in Fig. 1 and Fig. 2 could not be fabricated.

The refractive index and absorption coefficient plots of various materials tested during the project is shown in Fig. 4. The materials which were tested include HDPE, LDPE, PTFE, TPX, Nylon 6 and Polycarbonate. Point to note, all the materials studied were pure in their composition and no impurity was added to alter the material properties of these materials. However, in principle, impurity based variants of these materials are also available in the market and many can be used with commercial 3D printers but our main objective during the project was to design DOEs which can be manufactured
universally at any place. Therefore, keeping all this in mind, initially, as previously stated, HDPE was chosen because the refractive index was reasonably high in the range in which we envisioned the designs to operate, and the absorption coefficient was also close to 0. A more suitable candidate would have been LDPE due to its very high refractive index and equally low absorption coefficient but LDPE as a 3D printable material is not very standard and special process recipes needed to be developed to even make it suitable for printing in the first place. Even though initial efforts were made to develop a suitable fabrication process recipe, but later on achieving the desired resolution created a hindrance which ultimately led us to disregard LDPE. However, one very important takeaway from this study is that essentially all the materials barring Nylon 6 and Polycarbonate, can be used to fabricate the MDLs with the use of a suitable 3D printer and an appropriate process recipe. Impurity based variants can also be used but caution needs to be taken care that the optical constants of the material use does not change during the fabrication run otherwise the focusing performance of MDLs as shown in [15] can be “very” bad.

Fig. 4 The left panel plots the refractive index (n) of various materials suitable for 3D printing DOEs operating at THz frequencies (i.e., 0.25 THz – 0.75 THz). The right panel plots the corresponding absorption coefficients values (k) of the same.

Fig. 5 summarizes the simulated and measured intensity distributions for the 1D (Fig. MDL design shown in Fig 3(a). Simulations of the 1D MDL (f = 50 mm) predicts an average efficiency of ~91% whereas, the measured efficiency is about 85%. The measured point-spread functions, as seen in Fig. 5 are wider than the predicted distributions, mostly due to the slight resolution mismatch of the THz imager that was used, and the actual diffraction limited focal spot size of the THz beam. This could also be a reason as to why there is a decrease in measured efficiency compared to the predicted objective, in addition to common fabrication errors (which were again very prominent at least in the earlier MDL designs). Later on, some care was taken to
mitigate some of these issues arising from the fabrication runs but still a lot of them still remain unsolved. Shown in **Fig. 5(b)** are the convergence plots depicting the evolution of the efficiency with number of iterations; a much faster convergence (10x-100x) was observed for the optimization technique which was used when compared to the traditional DBS algorithm.

![Image](image_url)

**Fig. 5 (a)** Simulated and measured point-spread functions for 1D THz MDL operating at 0.3 THz. The top left panels depict the simulated and measured PSF as well as the calculated efficiency for designs with \( f = 50 \) mm. **(b)** The bottom left panels depict convergence plots of efficiency with number of iterations. **(c-e)** The right panels depict simulations illustrating the evolution of the focusing of the diffraction limited focal spot as a function of distance for designs with \( f = 20, 40, \) and 50 mm. The corresponding simulated and measured PSFs are depicted at the bottom of the right panels. In all cases, frequency is 0.3 THz.

The primary reason for the faster convergence cannot be wholly contributed to a faster algorithm. Use of an appropriate FoM metric is also needed to guarantee a faster convergence. This point is equally important to keep in mind while designing any MDL. A detailed discussion on this specific topic has been provided in a later section. But coming back to the main topic of discussion, **Fig. 5(c-d)** depict simulations illustrating the evolution of the focusing of the diffraction limited focal spot as a function of distance for designs with \( f = 20, 40, \) and 50 mm. The efficiency and FoM metric used in these designs were taken from [6-7]. The PSF results indicate a wider measured focal spot in contrast to the theoretical diffraction limited focal spot. A similar observation is also made for the 2D MDL design shown in **Fig. 6**. **Note:** The intensities shown in **Fig. 5(c-d)** as well as **Fig. 6(c-d)** are un-normalized intensities.
Fig. 6 (a) Simulated and measured point-spread functions for 2D THz MDL operating at 0.3 THz. The top left panels depict the simulated and measured PSF as well as the calculated efficiency for designs with $f = 50$ mm. (b) The bottom left panels depict convergence plots of efficiency with number of iterations. (c-e) The right panels depict simulations illustrating the evolution of the focusing of the diffraction limited focal spot as a function of distance for designs with $f = 20$, 40, and 50 mm. The corresponding simulated and measured PSFs are depicted at the bottom of the right panels. In all cases, frequency is 0.3 THz.

Fig. 6 summarizes the simulated and measured intensity distributions for the 1D (Fig. MDL design shown in Fig. 3(b). For the 2D MDL designs shown in Fig. 3(b), simulations predict an average efficiency of $\sim 93\%$. The measured efficiency in this case is about 92\%. The fact that the agreement between measurements and simulations is better for the 2D lens is attributed to the 2D design-space containing a much larger number of pixels, which in theory, translates into more degrees of freedom. The MDL designs shown in Fig. 3 were scrapped off because the measurement results could not be replicated in the later samples mainly due to imperfect fabrication. The major problem with all these “non-symmetric” MDL designs is repeatability. In contrast, “symmetric” MDLs are easier to manufacture at large scale with good repeatability in performance. However, it cannot be certainly ruled out that non-symmetric structures won’t work as well as symmetric structures at least from a performance perspective. At least till yet, there is no evidence for this statement to hold. Therefore, this is the main key takeaway here. A detailed exploratory study needs to be carried out to confirm the performance limits/thresholds for both “non-symmetric” and “symmetric” MDL. But even barring that the major challenge here is fabrication with precise control!
2.2. Symmetric THz MDLs
Apart from the fabrication complexity, the “non-symmetric” MDL designs are also computationally intensive. This was the reason why broadband “non-symmetric” MDL designs were never made. Initial demonstrations of “non-symmetric” MDLs are all narrowband but it should never be thought of that broadband “non-symmetric” MDL designs cannot be made rather the time and effort required to design such an MDL would be extremely time-consuming.

Fig. 7 Simulated and measured point-spread functions for 1D broadband THz MDL (0.3 THz – 0.6 THz). (a) The top left panels depict the calculated efficiencies for designs with f = 50 mm. (b) The bottom left panels depict theoretical diffraction limited and measured full-width at half-maximum (FWHM) of the focal spot as a function of frequency. (c-h) The right panels depict the simulated and measured PSFs.

Summarized in Fig. 7 and Fig. 8 are our broadband designs for both 1D and 2D MDLs. In the initial stages, most of our MDL design parameters were all kept very much the same. This was done to speed up the design time because in the initial stages it was quite unsure of that good MDL designs could be possible for all combinations of design parameters. As the convergence to a suitable design more often than not depends on a defining a suitable FoM rather than a faster or better optimization algorithm. For the broadband 1D THz MDL (Fig. 7(a)), the simulations predict an average efficiency of ~78% whereas, the measured efficiency was ~75%. Fig. 7(b) shows the measured focal spot in comparison to the theoretical diffraction limited focal spot; results indicate a 5-8% deviation between measured and theoretical values. The pixel height distribution for this broadband MDL is un-available at this time (since the pixel data file got lost!) but in terms of design parameters, the total number of rings (P) = 130, ring width (W) = 0.1
mm (for 1D), maximum pixel height ($H_{\text{max}}$) = 1.4 mm, minimum pixel height ($H_{\text{min}}$) = 0.1 mm, total length ($L$) = 26 mm, number of levels ($N$) = 14, focal length ($f$) = 50 mm and the numerical aperture (N.A.) = 0.2516. Fig. 7(c-h) depict the simulated and measured PSFs. Note: semi-analytic and simulated carry the same meaning.

Fig. 8 Simulated and measured point-spread functions for 2D broadband THz MDL (0.3 THz – 0.6 THz). (a) The top left panels depict the calculated efficiencies for designs with $f = 50$ mm. (b) The bottom left panels depict theoretical diffraction limited and measured full-width at half-maximum (FWHM) of the focal spot as a function of frequency. (c-h) The right panels depict simulated PSFs.

For its 2D counterpart (Fig. 8(a)), simulations predict an average efficiency of ~83% in contrast to a measured efficiency of ~81%. In this case too, the pixel height distribution for this broadband MDL is un-available at this time (since the pixel data file got lost!) but in terms of design parameters, the total number of rings ($P$) = 130, ring width ($W$) = 0.1 mm, maximum pixel height ($H_{\text{max}}$) = 1.4 mm, minimum pixel height ($H_{\text{min}}$) = 0.1 mm, total length ($L$) = 26 mm, number of levels ($N$) = 14, focal length ($f$) = 50 mm and the numerical aperture (N.A.) = 0.2516. Contrary to the previous case of “non-symmetric” MDL designs where the 2D designs had a pixel width ($W$) = 0.65 mm, the “symmetric” designs now could effectively be fabricated with a ring width ($W$) = 0.1 mm. This provided a huge improvement. Fig. 8(b) shows the measured focal spot in comparison to the theoretical diffraction limited focal spot; results indicate a much smaller deviation between measured and theoretical values as compared to its 2D analogue. We can simply attribute this to more degrees of design space freedom. Fig. 8(c-h) depict simulated PSFs. Note: semi-analytic and simulated carry the same meaning.
Fig. 9 (a-b) The top panels depict convergence plots of Figure of Merit Function (FoM) with number of iterations for narrowband THz MDL operating at 0.3 THz. (c-d) The bottom panels depict convergence plots of Figure of Merit Function (FoM) with number of iterations for narrowband THz MDL operating from 0.3 THz to 0.6 THz.

Fig. 9 shows a very interesting plot. The top panels i.e., Fig. 9(a-b) depict convergence plots of Figure of Merit Function (FoM) with number of iterations for narrowband THz MDL operating at 0.3 THz. Fig. 9(c-d) depict convergence plots of Figure of Merit Function (FoM) with number of iterations for narrowband THz MDL operating from 0.3 THz to 0.6 THz. The pixel height distribution and design parameters for these MDL designs are also un-available at this time but the “key takeaway” from this plot is that converging to a suitable solution within a certain iteration is never guaranteed under any condition. The choice and definition of a suitable FoM metric is central here. A rigorous proof of this statement cannot be given at this point since a detailed exploratory study needs to be carried out to confirm this as well. But pure optimization techniques will converge to the solution faster than conventional search algorithms. This is not something new or novel rather intuitive. But prior experience with MDL designs at least in the initial stages reveal that the trick to solving these kind of non-convex optimization problems depends a lot on the choice of important optimizable metrics rather than the algorithm itself. This fact should always be kept in mind! This is also one of the main reasons why numerous MDL papers [6-46] do not deal with a lot of discussion on the algorithmic implementation rather focusses much more on the results achieved.
2.3. THz Extended Depth of Focus (EDoF) MDLs
Similar to EDoF MDLs previously demonstrated in [11], an attempt was made to see if such EDoF MDLs can also be designed for other wavelength/frequencies bands. In terms of design parameters for the EDoF MDLs, the total number of rings (P) = 30, ring width (W) = 0.4 mm, maximum pixel height (Hmax) = 1 mm, minimum pixel height (Hmin) = 0.1 mm, diameter (D) = 24 mm, number of levels (N) = 11, focal length (f) = 50 mm right up to 2 m.

**Fig. 10 (a)** Ring height distribution of an Extended Depth of Focus (EDoF) THz MDL designed to operate at 0.3 THz. The top panel shows the (b) simulated z-propagation plot and the (c) simulated point-spread function (PSF) for the THz EDoF MDL at observation plane distance, d = 2000 mm. The bottom panel (d) shows the simulated PSFs at the rest of the observation plane distances.

**Fig. 10** shows the simulation results obtained for the EDoF MDL designed to operate at 0.3 THz. The top panels respectively show the ring height distribution in **Fig. 10(a)**, the z-propagation in **Fig. 10(b)**, and the PSF at the 2m observation distance in **Fig. 10(c)**. The bottom panel shows the PSFs at all the other observation planes. PSFs below 200 mm were not plotted because of the scale (since those PSFs would be too small to be observed on the plot). The optical micrograph of this EDoF MDL is not available at this time but **Fig. 11** depicts the measurement results which was obtained. One important point to note is that a Fresnel lens with a focal length = 50 mm was also designed and fabricated to compare the EDoF performance of the designed MDL with respect to the Fresnel lens. As will be explained later, the EDoF measurements in the THz regime are a lot more difficult to achieve in comparison to the visible wavelength band. Since the
power efficiency at very high distances from the MDL drops considerably, very high incident power sources are required. Unfortunately, we do not really have those!

![Image of measured PSFs for different distances]

**Fig. 11** The top panel depicts the measured PSFs of the same Extended Depth of Focus (EDoF) THz MDL design shown in Fig. 10 across the observation plane with distance $d$ ranging from 50 mm to 200 mm. (b) The bottom panel shows the measured PSFs of a Fresnel lens (with a focal length, $f = 50$ mm) across the same observation plane with distance $d$ ranging from 50 mm to 200 mm. Beyond 200 mm, the power was so less that the THz beam could not be aligned (or observed on the screen) to record the PSF measurements.

Due to this limitation, PSF measurements beyond 200mm was not possible as the power was too less, and the signal was too weak for the THz imager to detect anything. Apart, from this there was also a huge beam alignment issue. From the opt panel of **Fig. 11**, it can be seen that the recorded PSFs are not all center aligned. The cause for this issue is still unknown because such an issue was not observed in any of the visible EDoF MDLs [11]. But the key takeaway here is that EDoF MDLs like normal achromatic MDLs can be scaled to any wavelength/frequency band but measurements will probably be a lot more challenging. The Fresnel lens behaves how a normal THz Fresnel lens would i.e., the incident beam comes into focus and later diverges.

Apart from the issues highlighted above which still needs a lot of explanation, the fact that whether a broadband EDoF MDL could be designed still remains open. The project does not extend to broadband cases, but intuition and experience says that it can be designed. Moreover, the trade-off between efficiency v/s EDoF still needs to be explored to put upper bounds on its performance metrics. Finally, the fact that these EDoF MDLs behave similar to traditional axicons can make one wonder “What is the physics behind these?”. Unfortunately, this still remains an open question to answer.
2.4. Multi-Foci THz MDLs

Similar to the EDoF MDLs described in the previous section, another research direction which we wanted to explore but ultimately couldn’t is the design of multi-foci MDLs. Over and over again, almost all of the preliminary designs depicted in this manuscript mostly talk about designing MDLs in the THz regime is because of the fact that THz MDLs are easier to fabricate rather than 3D print (due to their larger super-wavelength feature sizes) than MDLs operating in the visible waveband.

Fig. 12 Schematic of a multi-foci THz MDL operating at (a) 0.3 THz and its corresponding (b) ring height distribution. The focal lengths are \( f_1 = 150 \text{ mm}, f_2 = 200 \text{ mm}, \) and \( f_3 = 250 \text{ mm}. \) (c) A broadband multi-foci THz MDL operating at three discrete THz frequencies of 0.3 THz, 0.4 THz and 0.5 THz is shown and its corresponding (d) ring height distribution. This MDL too has focal lengths, \( f_1 = 150 \text{ mm}, f_2 = 200 \text{ mm}, \) and \( f_3 = 250 \text{ mm}. \)

Fig. 12 shows the schematic of such a multi-foci THz MDL operating under both a narrowband (Fig 12(a)) and broadband (Fig. 12(c)) condition. Now, multi-foci DOEs are not something that is extremely new to the diffractive optics community. They have been around for almost the same amount of time as the DOEs themselves. However, the main aim here was a bit different. The idea was to design MDLs with focal lengths at “uneven” observation plane distances. Ideally one can argue here that a hologram also technically does the same thing. That is true and we do not deny this but the key challenge here was to validate whether one can improve the performance metrics upon the existing state-of-the-art DOE devices. In the initial stages, the idea of “uneven” interfocal length distances was dropped (as shown in Fig. 12) rather MDLs with “even” interfocal length distances were adopted i.e., \( f_1 = 150 \text{ mm}, f_2 = 200 \text{ mm}, \) and \( f_3 = 250 \text{ mm}. \) I know that throughout this manuscript, I use a lot of “non-technical” words probably one would
not expect from a person working in diffractive optics for almost half a decade! But this manuscript is intended to be read by a very general audience, so I leave out all the technical jargons.

Fig. 13 A multi-foci THz MDL operating at 0.3 THz and its corresponding ring height distribution is shown in the top panel along with the simulated PSFs at are $f_1 = 150$ mm, $f_2 = 200$ mm, and $f_3 = 250$ mm. The bottom panel shows the 3D printed MDL. The experimental results are not available at this time.

But coming back to the main discussion, Fig. 12(b) and Fig. 12(d) does show that MDLs with such functionalities can be designed which is corroborated with the results shown in Fig. 13 for the MDL design operating at a single frequency of 0.3 THz. In fact, the MDL designs were even fabricated in PLA but ultimately, the results could not be verified. Similar to the previous EDoF THz MDL case, the measurements were plagued with the curse of low incident power sources. An attempt was made later on to redesign with shorter focal length distances, but time ran out.

To wrap up, in terms of design parameters for the Multi-foci MDLs, the total number of rings ($P$) = 35, ring width ($W$) = 0.4 mm, maximum pixel height ($H_{\text{max}}$) = 1.5 mm (for narrowband) and 2 mm (for broadband), minimum pixel height ($H_{\text{min}}$) = 0.1 mm, diameter ($D$) = 28 mm, and number of levels ($N$) = 11. The next section is going to shed more light on the real objective behind the designs of these MDLs.
2.5. Broadband THz Multi-plane Projection Holograms

Continuing with the discussion from the previous section, the main focus for designing multi-foci MDLs was to focus the incident beam at “uneven” inter-focal length distances. Now, “focus” as a term can have many different meanings in different context. But if we simply consider a case, whereby we transfer information from the lens plane to the observation plane then this means that we are actually designing a “hologram”. Now, a lens can, in principle, be thought of as a hologram. This intuition is not straight-forward but if we look closely then one can see that if the constraint of “symmetry” (rings) is removed from an MDL design then the design space becomes large, and one can design any pre-defined intensity pattern. Again, this is not something extremely new. DOEs are already available in the commercial market as pattern generators, beam shapers, diffusers etc.

![Diagram of multi-plane projection holograms](image)

**Fig. 14** Schematic of a proposal to design multi-plane projection holograms based on multi-level pixelated structures or BDOEs which can effectively function as cryptic ciphers by embedding the original text within the structures themselves. **(a)** shows an incomplete designs operating at 0.2 THz and **(b)** shows an incomplete design at 0.3 THz.
But if we go back to our previous arguments, then even with the “symmetry” (rings) constraint, one should ideally be able to generate multi-foci MDL with “uneven” inter-focal length distances. Some initial MDL designs were achieved but the results (even in simulation) were so poor that they had to be dropped. In the interest of time, a detailed study could not be undertaken to understand why this happened? This was one of the failed areas of the project.

Fig. 15 (a) Schematic of a proposal to design multi-plane projection holograms based on multi-level pixelated structures or BDOEs with observation plane distance $d_1 = 50$ mm and $d_2 = 75$ mm operating at THz wavelengths 1 mm (0.3 THz), 0.5 mm (0.6 THz), and 0.6 mm (0.5 THz). (b) A happy smiley face which is set as the target image and (c) its corresponding pixel height distribution. The features could not be fabricated at the current 3D printing facility.

So, ultimately, “symmetry” (rings) constraint was dropped along with the name “MDL” and it was later decided to call these structures “Broadband Diffractive Optic Element (BDOE)”.

Fig. 14 and Fig. 15 shows the schematics developed during the project which conveys the same thought that had been previously discussed in this section. Fig. 15(c) shows one such design which projects a happy smiley face with observation plane distance $d_1 = 50$ mm and $d_2 = 75$ mm operating at THz wavelengths of 1 mm (0.3 THz), 0.5 mm (0.6 THz), and 0.6 mm (0.5 THz). Complicated target images could also be taken but as an initial proof-of-concept demonstration, a simple binary valued target image was considered.
In terms of design parameters, the pixel width \( W \) = 0.65 mm, maximum pixel height \( H_{\text{max}} \) = 2.5 mm, minimum pixel height \( H_{\text{min}} \) = 0.1 mm, diameter \( D \) = 26 mm, and number of levels \( N \) = 11.

Fig. 16 Schematic of a proposal to design multi-plane projection holograms based on multi-level pixelated structures or BDOEs which can effectively function as cryptic ciphers by embedding the original text within the structures themselves. The decryption key are the operation parameters (hard keys) in “correct” order. The multilevel structures provide for complicated messages to be embedded due to higher degrees of design freedom thereby ushering a new era of THz optical cryptography. (a) and (b) are two such designs made but not experimentally tested.

Fig. 16 shows an interesting (rather ambitious) idea developed. This idea could not really be executed because of time. Moreover, for this idea to work, all the MDL and BDOE designs discussed earlier needed to work. The idea builds on the premise that BDOEs could efficiently project intensity patterns at the pre-defined observation planes depending on the specific wavelength (or frequency) of excitation. These operation parameters can then ideally be used as “hard” decryption key in “correct” order to reveal the projected intensity (in this case, a written message). Given the fact that the design space and the space-bandwidth product for these multilevel structures is so huge that embedding complicated messages would not be extremely difficult. However, in spite of this, fabrication complexity does indeed make the manufacturing difficult. In fact, as shown in Fig. 16(a) and Fig. 16(b), two separate designs were achieved but neither fabrication nor experiments could be carried out to fruition. Speaking of design parameters, they were similar to the BDOE design of Fig. 15. In the first design (Fig. 16(a)), the operating wavelength was 1 mm (0.3 THz) and \( d_1 = 50 \) mm, \( d_2 = 75 \) mm, and \( d_3 = 115 \) mm. In the second design (Fig. 16(b)), the operating wavelengths 1 mm (0.3 THz), 0.5 mm (0.6 THz), and 0.6 mm (0.5 THz). The observation plane distance \( d_1 = 50 \) mm, \( d_2 = 75 \) mm, and \( d_3 = 115 \) mm.
2.6. Integral Imaging with MDLs
Preliminary results (including design parameters) for this MDL design and its associated micro lens array was published in [16]. Apart from super-resolution, integral imaging experiments were also carried out. It was unfortunate that these results did not form a part of [16] since the reviewers asked for a separate paper on this and time constraints did not permit such. Nonetheless, the results

![Fig. 17 (a)](image1.png) Optical micrograph of a single fabricated MDL within the (b) entire micro lenses array which consists of 40 x 40 micro-MDLs. (c) The camera assembly which shows the image sensor placed at the focal length of the micro lens array. (d) The image of three pebbles taken with this camera. The image is pixelated because each micro lens now acts as a single pixel (relative positions of whom will later be used to reconstruct the integral image and construct the depth map and perspective projection). (e) The corresponding white light PSF is also recorded which is later going to be used to construct the integral image. (f) The reconstructed integral image used to generate the perspective (or depth) view. The shift in light intensity across 2 frames of the perspective motion video shows the shift of light intensity gradient when the image is viewed from left to right. The reconstructed video is available at: [https://github.com/Sourangsu/Multilevel-Diffractive-Optics-based-Computational-Photography-Code/blob/main/videos/perspectiveAnimation_stSS15_uvSS10_aperMaskcirc.mp4](https://github.com/Sourangsu/Multilevel-Diffractive-Optics-based-Computational-Photography-Code/blob/main/videos/perspectiveAnimation_stSS15_uvSS10_aperMaskcirc.mp4)

**Fig. 17(f)** is self-explanatory as it typically shows the reconstructed integral image used to generate the perspective (or depth) view. The shift in light intensity across 2 frames of the perspective motion video shows the shift of light intensity gradient when the image is viewed from left to right. The reconstructed video from which the frames have been taken is available at following GitHub [link](https://github.com/Sourangsu/Multilevel-Diffractive-Optics-based-Computational-Photography-Code/blob/main/videos/perspectiveAnimation_stSS15_uvSS10_aperMaskcirc.mp4). But the key takeaway here is that MDLs similar to metalenses can to a huge extent portray the same multi-functional properties and even from a computational imaging/photography perspective a lot can be done and is still open to exploration.
2.7. Optimizing MDLs beyond DBS

Last but not the least, a section specifically dedicated to optimization algorithms for MDLs. Apart from the mDBS algorithm used to design majority of the MDLs, various genetic algorithms were also tried out. For example, particle swarm algorithm was also used during the initial phase of the work besides using the traditional DBS algorithm. A lot of time was never dedicated to developing algorithms because the project (or work) revolved around exploring the functionalities which can be enabled by the multilevel structures (i.e., MDLs or BDOEs) rather than developing a design framework. A lot of this design framework is mundane as shown in [6].

Fig. 18 Schematic of a proposed modification to the existing mDBS algorithm. The proposal was to check for FoM oscillations during the optimization process (non-improvement or minimal improvement in the FoM function) and update the FoM criterion accordingly. The modification was aimed to make the optimization escape the undesirable local minima solutions by providing a stochastic gradient jump.

Fig. 18 shows one such modification to the proposed mDBS algorithm which was ultimately scrapped off (an area of possible future exploration). The proposal was to check for FoM oscillations during the optimization process (non-improvement or minimal improvement) and update the FoM criterion accordingly. The modification was aimed to make the optimization escape the undesirable local minima solutions by providing a stochastic gradient jump. As previously stated, since the work mostly revolved around the results rather the cause, algorithm was never the central theme.
3. Conclusion

This manuscript provides an account of some of the unpublished results on multilevel diffractive optics during the 2016-2020 period. To the best of the knowledge of the authors, the results are authentic but still remained unpublished mainly due to time constraints and the need of additional experiments (at the request of the peer-reviewers) which could not be performed.
References